

# Relaxation time ansatz and bulk viscosity of hadron matter

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The bulk viscosity is calculated for hadron matter produced in heavy-ion collisions, being described in the relaxation-time approximation within the relativistic mean-field-based model with scaled hadron masses and couplings. We show how different approximations used in the literature affect the result. Numerical evaluations of the bulk viscosity with three considered ansatze deviate not much from each other confirming earlier results.

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**Introduction.** Recently, interest in the transport coefficient issue for hadronic and quark matter has essentially increased due to clarifying the role of viscosity in extraction of flow parameters from heavy-ion collisions, see review-article [1]. Viscosity coefficients in a weakly coupled scalar field theory at an arbitrary temperature can be evaluated directly from the first principles using expansion of the Kubo formulas in terms of ladder diagrams in the imaginary time formalism [2]. Unfortunately, similar analysis for more general cases is unavailable. Therefore, in order to evaluate transport coefficients in multi-component systems with a strong coupling between species, one often uses a kinetic approach. Thus, one can exploit the relaxation time approximation to the Boltzmann-like quasiparticle kinetic equations. The shear and bulk viscosities of the hadron and the quark-gluon plasma phases of strongly interacting matter at finite temperature and baryon density were evaluated, see Refs. [3–5]. In [6–8], a similar analysis was performed for purely gluon matter. Besides the use of the relaxation time approximation, one needs to do some extra assumptions in order to proceed further. In [7], we studied how different ansatze used in the literature affect the result for shear and bulk viscosities for gluon matter. We found that the result for the shear viscosity was robust with respect to different ansatz reductions, whereas the value of the bulk viscosity significantly depends on them. In this note we study how different approximations used in the literature affect the result for the bulk viscosity of the hadron matter.

**Model equations.** As in [4], we describe the hadron phase in terms of the quasiparticle relativistic mean-field (RMF) based model with the scaling hadron mass-couplings (SHMC) successfully applied earlier to the description of heavy-ion collision reactions [9], which is an extension of the model [10] applied there for the cold dense hadron matter. Bearing in mind an application to heavy ion collisions we deal with a RMF-based model of iso-symmetric non-equilibrium hadron matter with  $\sigma$ - and  $\omega$ -meson mean fields and in contrast with ordinary RMF models we assume that not only baryon but also other hadron masses might depend on the  $\sigma$ -meson mean field. Also excitations emerging from the  $\sigma$  and  $\omega_0$  mean

fields are incorporated. We study the system with zero net strangeness and we use the same hadron set, as in [4, 9]. Considering small deviations from local equilibrium we keep only first-order gradient terms. Further details can be found in mentioned works [4, 9].

We start with general expressions for the baryon/strangeness 4-current and the energy-momentum tensor densities

$$J_{B,S}^\mu = \sum_a t_a^{B,S} \int d\Gamma_a \frac{p_a^\mu}{E_a} f_a(x, \vec{p}), \quad (1)$$

$$T^{\mu\nu} = \sum_a \int d\Gamma_a \frac{(p_a^\mu + X_a^\mu) p_a^\nu}{E_a} f_a(x, \vec{p}) + T_{\text{MF}}^{\mu\nu}. \quad (2)$$

Here  $d\Gamma_a = d_a \frac{d^3p}{(2\pi)^3}$ ,  $p_a^\mu = (E_a, \vec{p})$ ,

$$E_a = \sqrt{\vec{p}^2 + m_a^{*2}(\sigma)}, \quad X_a^\mu = X_a^0(\sigma, \omega_0) \delta^{\mu 0}, \quad (3)$$

$t_a^B, t_a^S$  are the baryon and strange charges of the  $a$ -hadron (antiparticles are included),  $d_a$  is a degeneracy factor,  $m_a^*(\sigma)$  is an effective mass,  $f_a$  is a quasiparticle distribution function,  $\sigma$  and  $\omega_0$  are mean scalar and vector meson fields,  $\omega^\mu = (\omega_0, \vec{0})$ ,  $X_a^0 \sim \omega_0$ ,

$$T_{\text{MF}}^{\mu\nu} = g^{\mu\nu} [U(\sigma) - V(\sigma, \omega)], \quad (4)$$

$$U(\sigma) = \frac{m_\sigma^{*2}(\sigma) \sigma^2}{2} + U_{\text{NL}}(\sigma),$$

$$V(\sigma, \omega) = \frac{m_\omega^{*2}(\sigma) \omega_0^2}{2},$$

$U_{\text{NL}}(\sigma)$  is the non-linear potential of the  $\sigma$  field.

Conservation laws of the baryon/strangeness 4-current and of the energy-momentum tensor densities are read as

$$\partial_\mu J_{B,S}^\mu = 0, \quad \partial_\mu T^{\mu\nu} = 0. \quad (5)$$

We assume that the quasiparticle distribution functions  $f_a$  obey a set of kinetic equations

$$E_a^{-1} p_a^\mu \partial_\mu f_a - \nabla E_a \nabla_p f_a = \text{St} f_a, \quad (6)$$

where  $\text{St} f_a$  is the collision term for the given species satisfying the conditions

$$\sum_a t_a^{B,S} \int d\Gamma_a \text{St} f_a = 0, \quad \sum_a \int d\Gamma_a \varepsilon_a \text{St} f_a = 0, \quad (7)$$

with the quasiparticle energy  $\varepsilon_a = E_a + X_a^0$ .

The collision term is zero for the local equilibrium distribution,  $\text{St} f_a^{\text{1.eq.}} = 0$ , where

$$f_a^{\text{1.eq.}}(E_a^{\text{1.eq.}}, \vec{p}, x^\mu) = \left\{ e^{[E_a^{\text{1.eq.}} - \vec{p} \cdot \vec{u}(x^\mu) - \mu_a^*(x^\mu)]/T(x^\mu)} \pm 1 \right\}^{-1}, \quad (8)$$

with  $+$  for fermions,  $-$  for bosons,

$$\begin{aligned} E_a^{\text{1.eq.}} &= E_a(\sigma^{\text{1.eq.}}), \\ \mu_a^*(x^\mu) &= t_a^B \mu_B(x^\mu) + t_a^S \mu_S(x^\mu) - X_a^0(\sigma^{\text{1.eq.}}, \omega_0^{\text{1.eq.}}), \\ \sigma^{\text{1.eq.}} &= \sigma^{\text{1.eq.}}(T, \mu_B, \mu_S), \quad \omega_0^{\text{1.eq.}} = \omega_0^{\text{1.eq.}}(T, \mu_B, \mu_S), \\ \mu_B, \mu_S &\text{ are the baryon and strangeness chemical potentials, and the four-velocity of the frame is } u^\mu \simeq [1, \vec{u}(x^\mu)] \text{ for } |\vec{u}| \ll 1. \end{aligned}$$

Applying the second Eq. (5) for  $\nu = 0$  and (7), and using (2) we derive the self-consistency conditions:

$$\begin{aligned} \frac{\partial V}{\partial \omega_0} &= \sum_a \int d\Gamma_a f_a \frac{\partial \varepsilon_a}{\partial \omega_0}, \\ \frac{dU}{d\sigma} - \frac{\partial V}{\partial \sigma} &= - \sum_a \int d\Gamma_a f_a \frac{\partial \varepsilon_a}{\partial \sigma}. \end{aligned} \quad (9)$$

For the equilibrium system these equations coincide with the conditions of maximum pressure  $\frac{\partial P}{\partial \omega_0} = 0$ ,  $\frac{\partial P}{\partial \sigma} = 0$ , where pressure  $P = \frac{1}{3} T_{ii}^{\text{1.eq.}}$ . Latin index  $i = 1, 2, 3$ .

In the general case it is impossible to solve the Boltzmann kinetic equations for the strongly interacting multi-hadron system appearing in the course of heavy ion collisions. However the collision term is greatly simplified in the so called relaxation time approximation, or better to say, in the relaxation time ansatz. *We will use the expansion near the local equilibrium state*

$$\text{St} f_a = -\delta f_a / \tau_a, \quad \delta f_a = f_a - f_a^{\text{1.eq.}}(E_a^{\text{1.eq.}}), \quad (10)$$

where  $\tau_a$  are in general the energy dependent quantities, i.e.  $\tau_a = \tau_a(E_a^{\text{1.eq.}})$ . These values can be evaluated from the cross sections of particle-particle interactions.

Since the expression for the shear viscosity,  $\eta$ , is easily recovered [4] within the relaxation time approximation and one needs no extra assumptions for that, below we study only the bulk viscosity,  $\zeta$ .

**Bulk viscosity.** The bulk viscosity is defined as the coefficient entering into the variation of  $T^{ii}$  in the local rest frame:

$$\delta T^{ii} / 3 = -\zeta \nabla \cdot \vec{u}, \quad (11)$$

and variations of the baryon/strange charge and the energy density should satisfy the so called Landau-Lifshitz conditions

$$\delta J_{B,S}^0 = \sum_a t_a^{B,S} \int d\Gamma_a \delta f_a = 0, \quad (12)$$

$$\delta T^{00} = \sum_a \int d\Gamma_a \varepsilon_a \delta f_a = 0. \quad (13)$$

From the Boltzmann equations within the relaxation time approximation we find

$$\delta f_a [\nabla \cdot \vec{u}] = \left[ \tau_a Q_a(\vec{p}^2) \frac{f_a(1 \mp f_a)}{T} \right]^{\text{1.eq.}} \nabla \cdot \vec{u}, \quad (14)$$

where

$$\begin{aligned} -Q_a(\vec{p}^2) &= \frac{\vec{p}^2}{3E_a} + \left( \frac{\partial P}{\partial \epsilon} \right)_{n_B, n_S} \\ &\times \left( T \frac{\partial \varepsilon_a}{\partial T} + \mu_B \frac{\partial \varepsilon_a}{\partial \mu_B} + \mu_S \frac{\partial \varepsilon_a}{\partial \mu_S} - \varepsilon_a \right) \\ &+ \left( \frac{\partial P}{\partial n_B} \right)_{\epsilon, n_S} \left( \frac{\partial \varepsilon_a}{\partial \mu_B} - t_a^B \right) \\ &+ \left( \frac{\partial P}{\partial n_S} \right)_{\epsilon, n_B} \left( \frac{\partial \varepsilon_a}{\partial \mu_S} - t_a^S \right), \end{aligned} \quad (15)$$

and  $\epsilon = T_{00}^{\text{1.eq.}}$ , cf. [3, 4].

**Ansatz of Refs. [3, 4].** Following the line sketched in Ref. [3], performing variations in Ref. [4] *we did not vary quantities which may depend on the distribution function only implicitly*, such as  $E_a$ . This approximation is well satisfied for non-relativistic systems, see [7]. Although the validity of this approximation becomes questionable in the application to relativistic systems, its use allows one to essentially simplify calculations for the bulk viscosity, which is important in the case of a complicated system of many strongly interacting particle species. Therefore, in [4] we considered it as an additional ansatz. Then the expression for the value  $\delta T^{ii}$  looks very simple

$$\delta T^{ii}[\delta f_a] = \sum_a \int d\Gamma_a \frac{\vec{p}^2}{E_a} \delta f_a, \quad (16)$$

since  $E_a$  values are not varied. Using (16) we arrived at the following expression for the bulk viscosity:

$$\zeta = -\frac{1}{T} \sum_a \left\langle \tau_a Q_a(\vec{p}^2) \frac{\vec{p}^2}{3E_a} \right\rangle. \quad (17)$$

Here we introduced the notation

$$\langle \Phi_a(\vec{p}) \rangle = \int d\Gamma_a [\Phi_a(\vec{p}) f_a(1 \pm f_a)]^{\text{1.eq.}}. \quad (18)$$

As in [3], *we have just assumed the validity of the Landau-Lifshitz conditions (12) and (13)*. Using the latter condition we found in [4] the expression for the bulk viscosity

$$\zeta_{\text{Ref. [4]}} = \frac{1}{3T} \sum_a \left\langle \tau_a Q_a(\vec{p}^2) \left( \frac{m_a^{*2}}{E_a} + X_a^0 \right) \right\rangle. \quad (19)$$

One should note that relations (12), (13) might not be fulfilled, until some additional conditions were not imposed, see below. Since we did not impose these extra conditions, the use of the Landau-Lifshitz conditions can be considered as an additional ansatz.

**Ansatz of the given work.** We again consider the relaxation time ansatz. But now we avoid additional two assumptions used in [3, 4]. To derive the expression for the bulk viscosity, we follow the procedure sketched in [7] for gluons (see Sec. III.A there).

First, from (9), (10) we find variations of mean fields

$$\begin{pmatrix} \delta\sigma \\ \delta\omega_0 \end{pmatrix} = \begin{pmatrix} \Lambda_\sigma & \Lambda_{\sigma\omega} \\ \Lambda_{\sigma\omega} & \Lambda_\omega \end{pmatrix} \begin{pmatrix} \sum_a \int d\Gamma_a \frac{\partial \varepsilon_a}{\partial \sigma} \delta f_a \\ \sum_a \frac{\partial X_a^0}{\partial \omega_0} \int d\Gamma_a \delta f_a \end{pmatrix}, \quad (20)$$

where

$$\Lambda_\sigma \Lambda = \sum_a \frac{\partial^2 X_a^0}{\partial \omega_0^2} n_a - \frac{\partial^2 V}{\partial \omega_0^2}, \quad (21)$$

$$\Lambda_{\sigma\omega} \Lambda = \frac{\partial^2 V}{\partial \omega_0 \partial \sigma} - \sum_a \frac{\partial^2 X_a^0}{\partial \omega_0 \partial \sigma} n_a,$$

$$\begin{aligned} \Lambda_\omega \Lambda &= \frac{d^2 U}{d\sigma^2} - \frac{\partial^2 V}{\partial \sigma^2} + \sum_a \left( \frac{dm_a^*}{d\sigma} \right)^2 \int d\Gamma_a \frac{\vec{p}^2}{E_a^3} f_a^{\text{1.eq.}} \\ &+ \sum_a \frac{d^2 m_a^*}{d\sigma^2} \rho_a^{sc} + \sum_a \frac{\partial^2 X_a^0}{\partial \sigma^2} n_a, \end{aligned}$$

$$\Lambda = (\Lambda \Lambda_{\sigma\omega})^2 - (\Lambda \Lambda_\sigma)(\Lambda \Lambda_\omega), \quad (22)$$

$n_a = \int d\Gamma_a f_a$  is the number density of the particle species “ $a$ ” and  $\rho_a^{sc} = \int d\Gamma_a \frac{m_a^*}{E_a} f_a$  is the scalar density. All integrals in matrix  $\Lambda$  are calculated in the rest reference frame of the fluid with the local equilibrium distribution function. With the help of these expressions the variation  $\delta T^{ii}$  can be expressed as

$$\delta T^{ii}[\delta f_a] = 3 \sum_a \int d\Gamma_a F_a(\vec{p}^2) \delta f_a, \quad (23)$$

with

$$F_a(\vec{p}^2) = \frac{\vec{p}^2}{3E_a} - K_\sigma \frac{\partial \varepsilon_a}{\partial \sigma} - K_\omega \frac{\partial X_a^0}{\partial \omega_0}, \quad (24)$$

$$K_\sigma = \kappa \Lambda_\sigma - \frac{\partial V}{\partial \omega_0} \Lambda_{\sigma\omega},$$

$$K_\omega = \kappa \Lambda_{\sigma\omega} - \frac{\partial V}{\partial \omega_0} \Lambda_\omega,$$

$$\kappa = \frac{dU}{d\sigma} - \frac{\partial V}{\partial \sigma} + \frac{1}{3} \sum_a m_a^* \frac{dm_a^*}{d\sigma} \int d\Gamma_a \frac{\vec{p}^2}{E_a^3} f_a^{\text{1.eq.}}.$$

In the relaxation time approximation, which we exploit in the given paper, using (14) we present the Landau-Lifshitz conditions (12), (13) as

$$\sum_a t_a^{B,S} \langle \tau_a Q_a(\vec{p}^2) \rangle = 0, \quad (25)$$

$$\sum_a \langle \tau_a \varepsilon_a Q_a(\vec{p}^2) \rangle = 0. \quad (26)$$

If these relations are not fulfilled with the particular distribution (14), we may still fulfill them doing the shift

$$\tau_a Q_a(\vec{p}^2) \rightarrow \tau_a Q_a(\vec{p}^2) + y^B t_a^B + y^S t_a^S + x \varepsilon_a, \quad (27)$$

where  $x$  and  $y^{B,S}$  are some constants. These constants are associated with the conservation of the energy and the baryon and strange charges. Values of  $y^B$  and  $y^S$  are similar to baryon and strange chemical potentials. Since  $\mu_S \neq 0$  even for hadron matter with zero net strangeness, we cannot exclude the term  $y^S$ .

If one considers only elastic scattering of particles described by the exact Boltzmann collision term, the replacement (27) generates new solutions of the original Boltzmann equation, see [5]. However, one can show that for multi-particle systems considered within the relaxation time approximation even with energy-averaged values of  $\tau_a$ , the above replacement does not result in new solutions. Even for one species but with the energy dependent relaxation time  $\tau(E^{\text{1.eq.}})$  the replacement does not generate new solutions. Thus, *we actually fulfill the Landau-Lifshitz conditions at the price that the solutions of the Boltzmann equations with the collision terms (10) are spoiled*. The new collision terms are

$$\begin{aligned} \text{St} f_a &= -\tau_a^{-1} \left[ \delta f_a \right. \\ &+ \left. \frac{y^B t_a^B + y^S t_a^S + x \varepsilon_a}{T} f_a^{\text{1.eq.}} (1 \mp f_a^{\text{1.eq.}}) \nabla \cdot \vec{u} \right]. \end{aligned} \quad (28)$$

Therefore, the requirement of fulfillment of the conditions (25), (26) should be considered as an ansatz.

After performing the replacement (27) in the conditions (25), (26), we arrive at the system of linear equations for  $x$  and  $y$ . Finally, we obtain

$$\begin{aligned} \zeta &= -\frac{1}{T} \sum_a \langle \tau_a Q_a(\vec{p}^2) \\ &\times [F_a(\vec{p}^2) - \gamma \varepsilon_a - t_a^B \chi_B - t_a^S \chi_S] \rangle, \end{aligned} \quad (29)$$

where

$$\begin{aligned} \gamma J &= \begin{vmatrix} \sum_a \langle \varepsilon_a F_a(p) \rangle & a_{12} & a_{13} \\ \sum_a t_a^B \langle F_a(p) \rangle & a_{22} & a_{23} \\ \sum_a t_a^S \langle F_a(p) \rangle & a_{23} & a_{33} \end{vmatrix}, \\ \chi_B J &= \begin{vmatrix} a_{11} & \sum_a \langle \varepsilon_a F_a(p) \rangle & a_{13} \\ a_{12} & \sum_a t_a^B \langle F_a(p) \rangle & a_{23} \\ a_{13} & \sum_a t_a^S \langle F_a(p) \rangle & a_{33} \end{vmatrix}, \\ \chi_S J &= \begin{vmatrix} a_{11} & a_{12} & \sum_a \langle \varepsilon_a F_a(p) \rangle \\ a_{12} & a_{22} & \sum_a t_a^B \langle F_a(p) \rangle \\ a_{13} & a_{23} & \sum_a t_a^S \langle F_a(p) \rangle \end{vmatrix}, \end{aligned} \quad (30)$$

$$(a_{ij}) = \begin{pmatrix} \sum_a \langle \varepsilon_a^2 \rangle & \sum_a t_a^B \langle \varepsilon_a \rangle & \sum_a t_a^S \langle \varepsilon_a \rangle \\ \sum_a t_a^B \langle \varepsilon_a \rangle & \sum_a (t_a^B)^2 \langle 1 \rangle & \sum_a t_a^B t_a^S \langle 1 \rangle \\ \sum_a t_a^S \langle \varepsilon_a \rangle & \sum_a t_a^B t_a^S \langle 1 \rangle & \sum_a (t_a^S)^2 \langle 1 \rangle \end{pmatrix},$$

$$J = \det \|a_{ij}\|.$$

**Ansatz of Ref. [5].** Above we assumed that the original Boltzmann collision terms  $\text{St}f_a^{\text{leq.}}(E_a^{\text{leq.}}) = 0$ . However, at calculating the viscosity coefficients in the quasiparticle Fermi liquid theory one often uses [11] that the original Boltzmann collision terms  $\text{St}f_a^{\text{leq.}}(E_a) = 0$  also for  $E_a$ , being functionals of exact non-equilibrium distribution functions. This is so because the energy conservation  $\delta$ -function pre-factors depend on exact particle energies. Thus, *in the relaxation time approximation one can write*

$$\text{St}f_a = -\delta\tilde{f}_a/\tilde{\tau}_a, \quad \delta\tilde{f}_a = f_a - f_a^{\text{leq.}}(E_a), \quad (31)$$

and thereby

$$\delta f_a = \delta\tilde{f}_a + \frac{\partial f_a}{\partial \varepsilon_a} \left[ \frac{\partial \varepsilon_a}{\partial \sigma} \delta\sigma + \frac{\partial X_a^0}{\partial \omega_0} \delta\omega_0 \right]. \quad (32)$$

The relaxation time  $\tilde{\tau}_a$  is in general different from  $\tau_a$  introduced above. Note that due to the smallness of  $\delta f_a$ , one can consider here  $\tilde{\tau}_a = \tilde{\tau}_a(E_a^{\text{leq.}})$  as a function of the energy  $E_a^{\text{leq.}}$ .

The difference between the approach [5] and that exploited in section "Ansatz of the given work" is that following [5] we now express all variations through  $\delta\tilde{f}_a$  and  $\tilde{\tau}_a$  rather than through  $\delta f_a$  and  $\tau_a$ . Since  $E_a$  are now fixed and thus not varied, we get the same expression for the value  $\delta T^{ii}$  as (16) but now with the substitution  $\delta\tilde{f}_a$  instead of  $\delta f_a$ . Here one should note that in Section "Ansatz of [3, 4]" the quantity  $E_a[f_a]$  was not varied according to our ansatz, while now the expression for  $\delta T^{ii}[\delta\tilde{f}_a]$  becomes a fully correct relation. Within the relaxation time approximation using (31) we arrive at expression (17) with the only difference that  $\tau_a$  should be now replaced by  $\tilde{\tau}_a$ . Thus, performing similar calculations to those we have done in the previous section we rewrite now the Landau-Lifshitz conditions as

$$\sum_a \left\langle \tilde{\tau}_a Q_a \left( t_a^{B,S} - \frac{\partial \varepsilon_a}{\partial \mu_{B,S}} \right) \right\rangle = 0, \quad (33)$$

$$\sum_a \left\langle \tilde{\tau}_a Q_a \left( \varepsilon_a - T \frac{\partial \varepsilon_a}{\partial T} - \mu_B \frac{\partial \varepsilon_a}{\partial \mu_B} - \mu_S \frac{\partial \varepsilon_a}{\partial \mu_S} \right) \right\rangle = 0. \quad (34)$$

Making use of the shift

$$\tilde{\tau}_a Q_a(\vec{p}) \rightarrow \tilde{\tau}_a Q_a(\vec{p}) + y^B t_a^B + y^S t_a^S + x \varepsilon_a \quad (35)$$

and solving the corresponding system of linear equations for  $x$ ,  $y^B$  and  $y^S$  we find

$$\zeta_{\text{ChK}} = \frac{1}{T} \sum_a \langle \tilde{\tau}_a Q_a^2(\vec{p}) \rangle. \quad (36)$$

We note that, as in section "Ansatz of the given work", *we actually fulfill the Landau-Lifshitz conditions at the price of spoiling the solutions of the Boltzmann equations with the collision terms (31).*

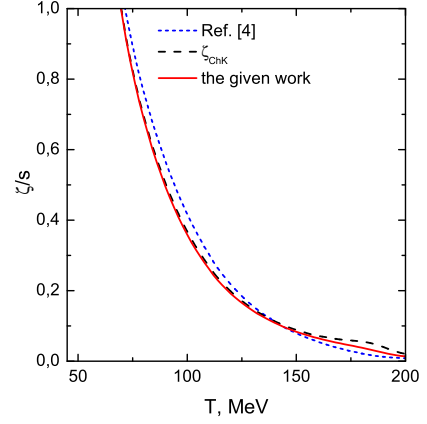


FIG. 1: (Color online) The ratio of the bulk viscosity to the entropy density as a function of temperature at  $\mu_B = 0$ .

Eq. (36) formally coincides with that obtained in [5] but new terms depending on  $\mu_{B,S}$  are involved, cf. (15).

**Numerical results.** Details of calculations of the relaxation time  $\tau_a$  and of shear and bulk viscosities in our SHMC model can be found in [4]. Since in such a complicated multi-particle system, as we study, values  $\tau_a$  cannot be calculated microscopically but can only be evaluated using empirical values of the cross sections, we cannot distinguish  $\tau_a$  and  $\tilde{\tau}_a$  and therefore, as in [7], we consider them to be the same.

In Fig. 1, we show the bulk viscosity to the entropy density ratio at  $\mu_B = 0$  as a function of the temperature. The solid line is the calculation of the given work performed following Eq. (29). The long-dashed curve is the result of Eq. (36) derived following the ansatz [5]. The short-dashed curve is our old result [4] calculated following Eq. (19). We see that all three results (especially those calculated following Eqs. (29) and (36)) are close to each other for temperatures  $T \lesssim 150$  MeV. For higher temperatures deviations become more pronounced.

In Fig. 2, the ratio of the bulk viscosity to the entropy density is presented as a function of temperature for two values of the baryon density  $n_B = n_0$  and  $4n_0$ , where  $n_0 = 0.16 \text{ fm}^{-3}$  is the nuclear density at the saturation point. The notation is the same as in Fig. 1. The results are shown for the SHMC model (a) and for the ideal gas (IG) model with the same hadron set (b). We see that the curves calculated by Eq. (29) and Eq. (36) are closer to each other than to the curve calculated with Eq. (19). For the IG, the curves calculated following Eqs. (29) and (36) coincide. Also, these curves are close to those estimated according to Eq. (19) for  $T \gtrsim 100$  MeV. Comparing figures (a) and (b) we also see that the presence of the quasiparticle interaction is more significant for low temperatures ( $T \lesssim 100$  MeV) and it becomes less important for higher temperatures.

**Conclusion.** Due to the complexity of the hadron system formed in actual heavy-ion collisions, it is hard to calculate the viscosity coefficients from the first principles with the help of the Kubo formulas. One could

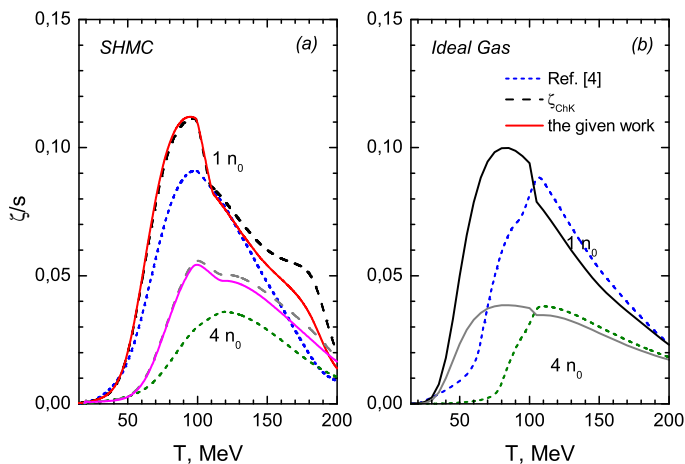


FIG. 2: (Color online) The ratio of the bulk viscosity to the entropy density as a function of temperature for two values of the baryon density  $n_B/n_0 = 1$  and 4 (from top to bottom).

use the Kadanoff-Baym kinetic equations for the hadron resonances to derive general expressions for the kinetic coefficients, but at present realistic calculations do not seem possible even in the relaxation time approximation [12]. Therefore, making use of the quasiparticle Boltzmann-like equations, being treated within the relaxation time approximation, can be considered as a forced step for practical evaluations of the kinetic coefficients in the given problem. The scaling hadron mass-couplings model of Ref. [9] is an appropriate model for the description of the equation of state of the hot and dense

hadron system. The knowledge of the latter is necessary in order to perform evaluations of the kinetic coefficients. However, even an application of simplified phenomenological expressions for the relaxation times of different species does not allow one to proceed in calculation of the bulk viscosity without doing additional assumptions, in particular the Landau-Lifshitz conditions should be fulfilled. However, these conditions cannot be satisfied on the class of solutions of the Boltzmann equations for our multi-component system treated within the relaxation time approximation, and additional ansätze are needed. Contrary, the result for the shear viscosity proves to be rather robust to these reductions, see [7].

We studied three ansätze previously used in the literature and derived three expressions for the bulk viscosity (19), (29) and (36), generalized to the case of nonzero chemical potentials  $\mu_{B,S}$ . Luckily, numerical evaluations performed following all three expressions deviate not much from each other and confirm earlier results [4–8]. Although these results can be considered only as rough estimations, among them the result, Eq. (29), seems to be the most theoretically justified. In order to perform more accurate calculations, one should go beyond the scope of the relaxation time approximation and to fulfill the Landau-Lifshitz conditions on the class of solutions of the kinetic equation. However, such calculations are much more involved than estimations presented in the given work and have not yet been carried out for multi-component systems with strong interactions.

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